P302 Photonics Fall 2010
Exam 1 Solutions

1. Writing the wave in the form $f(x \pm vt)$:

$$\psi(x,t) = A \exp\left[-a\beta^2\left(x + \frac{\gamma}{\beta}t\right)^2\right]$$

a) We see that $\boxed{v = \gamma/\beta}$ and the wave

b) is traveling along the negative x-axis.

2. a) Period $= T = \dfrac{1}{\text{frequency}} = \dfrac{1}{\nu}$

$$T = \frac{1}{10\,\text{MHz}} = \frac{1}{10^7/\text{s}} = 10^{-7}\,\text{s}$$

b) Wavelength $= \lambda = c/\nu$ ($v = c$ in free space)

$$\lambda = \frac{3\times10^8\,\text{m/s}}{10^7/\text{s}} = 30\,\text{m}$$

c) Energy $= \mathcal{E} = h\nu = (6.63\times10^{-34}\,\text{Js})\left(\frac{10^7}{\text{s}}\right)\left(\frac{1\,\text{eV}}{1.60\times10^{-19}\,\text{J}}\right)$

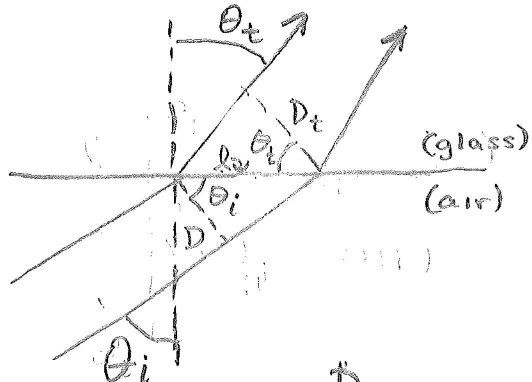
$$\mathcal{E} = 4.14\times10^{-8}\,\text{eV}$$

3. $$\#\text{waves} = \frac{\text{thickness}}{\text{wavelength in glass}}$$

$$= \frac{d}{\lambda_0/n} = \frac{1\,\text{cm}}{500\,\text{nm}/1.60}$$

$$= 3.19\times10^4$$

4.



Snell's Law:

$$1 \sin\theta_i = n \sin\theta_t$$

But $\cos\theta_i = \frac{D}{\ell}$ and $\cos\theta_t = \frac{D_t}{\ell}$

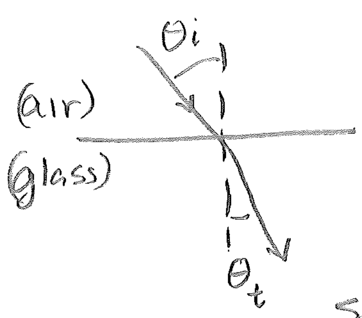
So $\frac{D_t}{D} = \cos\theta_t / \cos\theta_i$

But $\cos\theta_t = \sqrt{1 - \sin^2\theta_t} = \sqrt{1 - \frac{\sin^2\theta_i}{n^2}}$

$$= \frac{1}{n}\sqrt{n^2 - \sin^2\theta_i}$$

So $D_t = \frac{D\sqrt{n^2 - \sin^2\theta_i}}{n\cos\theta_i}$

5.



$$\sin\theta_i = n \sin\theta_t$$

But $\theta_t = \theta_i/2$

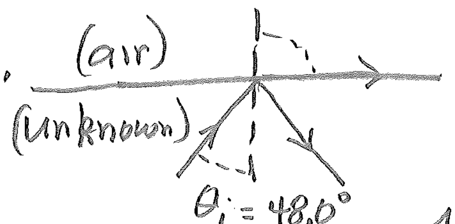
so $\sin\theta_i = n \sin\left(\frac{\theta_i}{2}\right)$

Also $\sin\theta_i = \sin 2\left(\frac{\theta_i}{2}\right) = 2\sin\frac{\theta_i}{2}\cos\frac{\theta_i}{2}$

So $\cos\frac{\theta_i}{2} = \frac{n}{2} = 0.85$

$$\theta_i = 2\cos^{-1}\left(\frac{n}{2}\right) = 63.6^\circ$$

6.

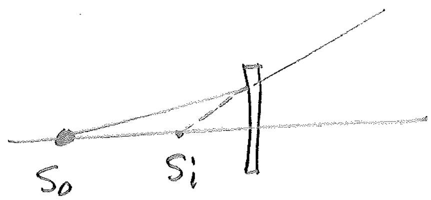


$\theta_i = \theta_{critical} = \theta_c$

so $n_u \sin\theta_c = 1 \sin 90°$

or $n_u = \frac{1}{\sin\theta_c} = 1.35$

7.



$S_o = 100$ cm

$S_i = -50$ cm

a) $\frac{1}{f} = \frac{1}{S_o} + \frac{1}{S_i}$ or $f = \frac{S_o S_i}{S_o + S_i} = \frac{(100\,cm)(-50\,cm)}{100\,cm - 50\,cm}$

or $f = -100$ cm

b) Image is on left side (virtual image)

c) Image is erect

d) Transverse magnification, $M_T = \frac{y_i}{y_o} = -\frac{S_i}{S_o}$

so $y_i = -y_o \frac{S_i}{S_o} = -2\,mm\left(\frac{-50\,cm}{100\,cm}\right)$

$y_i = 1\,mm$

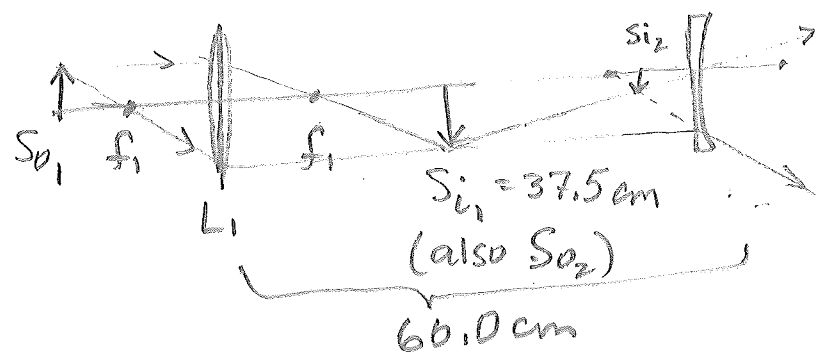
8. With only lens 1, find it's image:

$f_1 = +15.0\,cm$,

$$\frac{1}{S_{i_1}} = \frac{1}{f_1} - \frac{1}{S_{o_1}} \Rightarrow S_{i_1} = \frac{S_{o_1} f_1}{S_{o_1} - f_1}$$

$$S_{i_1} = \frac{(25.0\,cm)(+15.0\,cm)}{25.0\,cm - 15.0\,cm} = 37.5\,cm$$

Now use this image as the object for lens 2:



$S_{o_2} = 60.0\,cm - S_{i_1} = 22.5\,cm$ This is a real object

Apply lens equation again:

$$\frac{1}{f_2} = \frac{1}{S_{o_2}} + \frac{1}{S_{i_2}} \Rightarrow S_{i_2} = \frac{S_{o_2} f_2}{S_{o_2} - f_2} = -9\,cm$$

a) The image is at $S_{i_2} = -9\,cm$, it is 9 cm to the left of lens 2.

b) Image is virtual and inverted (wrt original object)